Independent Schools Examinations Board

COMMON ENTRANCE EXAMINATION AT 11+ AND 13+

COMMON ACADEMIC SCHOLARSHIP EXAMINATION AT 13+

MATHEMATICS SYLLABUS

(Revised Summer 2014 and reviewed Autumn 2014 for first examination in Autumn 2016)

INTRODUCTION

The curriculum which pupils follow at school should not be restricted by this examination syllabus which is intended to be used simply as a guide. It is presumed that, as in all good practice, teachers will, where it is appropriate for their pupils, teach beyond the syllabus. The syllabus reflects the initiatives in the National Curriculum published in September 2013 (updated July 2014). It is assumed that all teachers of mathematics will be familiar with the relevant publications. At 11+, it is expected that pupils will already be familiar with the skills and knowledge of the National Curriculum key stage 1 programmes of study and the programmes of study for Years 3 and 4, published September 2013, prior to starting the 11+ syllabus. At 13+, it is expected that pupils will be familiar with the key stage 2 programmes of study.

AIMS

A course leading to these examinations should:

(i) encourage breadth of experience in the development of mathematical skills without in any way prejudicing thorough grounding;

(ii) encourage the development of investigative thinking and the application of mathematical knowledge to unfamiliar problems.

ASSESSMENT OBJECTIVES

11+

Candidates should be familiar with the skills and knowledge of the National Curriculum key stage 2 programmes of study. The principal focus of mathematics teaching at 11+ is:

- to ensure that pupils extend their understanding of the number system and place value to include larger integers
- to develop the connections which pupils make between multiplication and division with fractions, decimals, percentages and ratio
- to develop pupils' ability to solve a wider range of problems, including increasingly complex properties of numbers and arithmetic, and problems demanding efficient written and mental methods of calculation
- to introduce pupils to the language of algebra as a means for solving a variety of problems
- to consolidate and extend knowledge developed in number in geometry and measure; to ensure that pupils classify shapes with increasingly complex geometric properties and that they learn the vocabulary they need to describe them
- to ensure that pupils are fluent in written methods for all four operations, including long multiplication and division, and in working with fractions, decimals and percentages
- to ensure that pupils read, spell and pronounce mathematical vocabulary correctly

13+

Candidates should be familiar with most of the skills and knowledge of the National Curriculum key stage 3 programmes of study.

During key stage 3, candidates build on the skills which they developed in key stage 2 and increasingly make connections between different aspects of mathematics. Candidates should build on key stage 2 and connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also be able to apply their mathematical knowledge in science, geography, computing and other subjects.

Working mathematically

Through the mathematics content, pupils should be taught to:

Develop fluency

- consolidate their numerical and mathematical capability from key stage 2 and extend their understanding of the number system and place value to include decimals, fractions, powers and roots
- select and use appropriate calculation strategies to solve increasingly complex problems
- use algebra to generalise the structure of arithmetic, including formulating mathematical relationships

- substitute values in expressions, rearrange and simplify expressions, and solve equations
- move freely between different numerical, algebraic, graphical and diagrammatic representations [for example, equivalent fractions, fractions and decimals, and equations and graphs]
- develop algebraic and graphical fluency, including understanding linear and simple quadratic functions
- use language and properties precisely to analyse numbers, algebraic expressions, 2-D and 3-D shapes, probability and statistics

Reason mathematically

- extend their understanding of the number system; make connections between number relationships, and their algebraic and graphical representations
- extend and formalise their knowledge of ratio and proportion in working with measures and geometry, and in formulating proportional relations algebraically
- identify variables and express relations between variables algebraically and graphically
- make and test conjectures about patterns and relationships; look for proofs or counter-examples
- begin to reason deductively in geometry, number and algebra, including using geometrical constructions
- interpret when the structure of a numerical problem requires additive, multiplicative or proportional reasoning
- explore what can and cannot be inferred in statistical and probabilistic settings, and begin to express their arguments formally

Solve problems

- develop their mathematical knowledge, in part through solving problems and evaluating the outcomes, including multi-step problems
- develop their use of formal mathematical knowledge to interpret and solve problems, including financial mathematics
- begin to model situations mathematically and express the results using a range of formal mathematical representations
- select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems

SYLLABUS CONTENT

The key stage 2 and 3 programmes of study describe the skills, knowledge and understanding required.

11+

During key stage 2, candidates build on the skills which they developed in key stage 1. They extend their competence and confidence with number so that they move from security in counting to security in calculating with all four operations. They develop their ability to solve a wide range of problems, including increasingly complex properties of numbers and arithmetic, and problems demanding efficient written and mental methods of calculation. Candidates are introduced to the language of algebra as a means for solving a variety of problems. Teaching in geometry and measures should consolidate and extend knowledge developed in number. Teaching should also ensure that candidates classify shapes with increasingly complex geometric properties and that they learn the vocabulary they need to describe them. In Year 6, candidates should be fluent in written methods for all 4 operations, including long multiplication and division, and in working with fractions, decimals and percentages. They should read, spell and pronounce mathematical vocabulary correctly.

Topics are centred on the National Curriculum for key stage 2 with particular reference to the Year 5 and 6 programmes of study; because of the timing of the 11+ examinations in November and January, not all statements will be examined, but it is recommended that those topics not included in the examination syllabus should still be taught in Year 6.

13+

Candidates should be familiar with most of the skills and knowledge of the National Curriculum key stage 3 programmes of study.

A number of elements from the key stage 3 programmes of study have been excluded from the syllabus, in line with pupils sitting Common Entrance in Year 8, whereas key stage 3 continues through to Year 9. A small number of elements which have been traditionally examined in Common Entrance, but are not specified in the key stage 3 programmes of study, have been retained from the previous syllabus.

It is expected that all pupils will have experience of the concepts in the core Level 2 syllabus. However, the treatment of those concepts in the examination may differ between Levels 1 and 2, with Level 1 questions generally being more accessible.

At Level 3, some of the questions will require a more sophisticated understanding of the core syllabus. In addition, there will be a small number of additional elements and questions as indicated in the syllabus content.

SCHEME OF ASSESSMENT

At both stages of transfer, working and answers are to be shown on the question paper, and in most questions failure to show working may result in loss of marks.

11+

Candidates will be required to work one paper of 60 minutes' duration. All candidates should answer as many questions as possible. Whilst candidates should be familiar with the use of calculators, calculating aids may not be used in the examination.

13+

Candidates will be required to work two papers, one non-calculator and one calculator, each of 60 minutes' duration, and a mental test lasting up to 30 minutes. The non-calculator and calculator papers will be available at three levels: **Level 1**, **Level 2** and **Level 3**. Level 1 is aimed at those candidates who would typically score less than 35% on Level 2 papers. Level 2 will be taken by the majority of candidates. Level 3 will test the extended specification. The mental test will be common to all three levels. It will be recorded.

Scholarship

The Common Academic Scholarship Examination (90 minutes) will be based on the extended 13+ Common Entrance syllabus, as indicated in the Subject Content table.

11+ SYLLABUS CONTENT

In the syllabus below:

- the complete key stage 2 programmes of study for Years 5 and 6 are shown in the left hand column
- *elements of the programme shown in italics will* ***not*** *be examined* at Common Entrance, but it is recommended that they still be taught before the end of Year 6; including non-statutory work
- further guidance and clarification about what will be examined at 11+ is given in the right-hand column
- for more detailed information, preparatory schools are advised to look at the *National Curriculum in England: Mathematics Programmes of Study* document, https://www.gov.uk/government/publications/national-curriculum-in-england-mathematics-programmes-of-study published in September 2013 and updated in July 2014, including non-statutory notes and guidance

Subject content

National Curriculum descriptors	**11+ examination guidance**
Number Pupils should be taught to:	
number and place value – Year 5	
read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit	
count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000	
interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through zero	
round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000	
solve number and practical problems which involve all of the above	
read Roman numerals to 1000 (M) and recognise years written in Roman numerals	
number and place value – Year 6	
read, write, order and compare numbers to at least 10 000 000 and determine the value of each digit	
round any whole number to a required degree of accuracy	

National Curriculum descriptors	11+ examination guidance
use negative numbers in context, and calculate intervals across zero	
solve number and practical problems which involve all of the above	include multistep problems, using the four operations and any relevant skills from those listed
addition, subtraction, multiplication and division – Year 5	
add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)	
add and subtract numbers mentally with increasingly large numbers	
use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy	
solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why	
identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers	
know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers	
establish whether a number up to 100 is prime and recall prime numbers up to 19	
multiply numbers up to 4 digits by a one- or two-digit whole number using a formal written method, including long multiplication for two-digit numbers	
multiply and divide numbers mentally drawing upon known facts	include mental recall of multiplication and division facts up to 12 x 12
divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context	
multiply and divide whole numbers and those involving decimals by 10, 100 and 1000	
recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})	
solve problems involving addition, subtraction, multiplication and division including using their knowledge of factors and multiples, squares and cubes	

National Curriculum descriptors	11+ examination guidance
solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign	
solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates	
addition, subtraction, multiplication and division – Year 6	
multiply multidigit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication	
divide numbers up to 4 digits by a two-digit number using the formal written method of long division and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context	*only division by numbers up to 12 or multiples of 10 will be examined, but teaching of division by other two-digit numbers in Year 6 is advised*
divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context	*only division by numbers up to 12 or multiples of 10 will be examined, but teaching of division by other two-digit numbers in Year 6 is advised*
perform mental calculations, including with mixed operations and large numbers	
identify common factors, common multiples and prime numbers	
use their knowledge of the order of operations to carry out calculations involving the four operations	
solve addition and subtraction multistep problems in contexts, deciding which operations and methods to use and why	
solve problems involving addition, subtraction, multiplication and division	
use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy	
fractions (including decimals and percentages) – Year 5	
compare and order fractions whose denominators are all multiples of the same number	
identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths	
recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number [for example, $\frac{2}{5} + \frac{4}{5} + \frac{6}{5} = 2\frac{2}{5}$]	

National Curriculum descriptors	11+ examination guidance
add and subtract fractions with the same denominator and denominators which are multiples of the same number	
multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams	
read and write decimal numbers as fractions [for example, 0.71 = $\frac{71}{100}$]	decimals up to three decimal places
recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents	
round decimals with two decimal places to the nearest whole number and to one decimal place	
read, write, order and compare numbers with up to three decimal places	
solve problems involving numbers up to three decimal places	
recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator 100, and as a decimal	
solve problems which require knowing percentage and decimal equivalents of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{4}{5}$ and those fractions with a denominator of a multiple of 10 or 25	solve number, measure, money and practical problems involving listed fraction, decimal and percentage skills
fractions (including decimals and percentages) – Year 6	
use common factors to simplify fractions; use common multiples to express fractions in the same denomination	
compare and order fractions, including fractions > 1	examination limited to fractions whose denominations are multiples of the same number, e.g. 5/6 and 11/12
add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions	*only fractions whose denominators are multiples of the same number will be examined and not mixed numbers, but teaching in Year 6 is advised*
multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, $\frac{1}{4}$ x $\frac{1}{2}$ = $\frac{1}{8}$]	
divide proper fractions by whole numbers [for example, $\frac{1}{3}$ ÷ 2 = $\frac{1}{6}$]	*this will not be examined, but teaching in Year 6 is advised*
associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, $\frac{3}{8}$]	*this will not be examined, but teaching in Year 6 is advised*

National Curriculum descriptors	11+ examination guidance
identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places	*this will not be examined, but teaching in Year 6 is advised*
multiply one-digit numbers with up to two decimal places by whole numbers	
use written division methods in cases where the answer has up to two decimal places	*this will not be examined, but teaching in Year 6 is advised*
solve problems which require answers to be rounded to specified degrees of accuracy	*this will not be examined, but teaching in Year 6 is advised*
recall and use equivalences between simple fractions, decimals and percentages, including in different contexts	limited to halves, quarters, fifths, tenths and hundredths
Ratio and Proportion Pupils should be taught to:	
Year 6 only	
solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts	
solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison	examination of percentages limited to multiples of 10, or 25% or 75%; but teaching in Year 6 of other percentages is advised
solve problems involving similar shapes where the scale factor is known or can be found	
solve problems involving unequal sharing and grouping using knowledge of fractions and multiples	
Algebra Pupils should be taught to:	
Year 6 only	
use simple formulae	will involve simple words and letters NB: the treatment of algebra will be largely informal with the emphasis upon understanding that letters can represent unknowns and variables
generate and describe linear number sequences	will include finding term-to-term rules
express missing number problems algebraically	

National Curriculum descriptors	11+ examination guidance
find pairs of numbers that satisfy an equation with two unknowns	
enumerate possibilities of combinations of two variables	
Measurement Pupils should be taught to:	
Year 5	
convert between different units of metric measure (for example, kilometre and metre; centimetre and metre; centimetre and millimetre; gram and kilogram; litre and millilitre)	
understand and use approximate equivalences between metric units and common imperial units such as inches, pounds and pints	
measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres	
calculate and compare the area of rectangles (including squares), and including using standard units, square centimetres (cm^2) and square metres (m^2) and estimate the area of irregular shapes	
estimate volume [for example, using 1-cm^3 blocks to build cuboids (including cubes)] and capacity [for example, using water]	
solve problems involving converting between units of time	including solving problems involving time and timetables
use all four operations to solve problems involving measure [for example, length, mass, volume, money] using decimal notation, including scaling	
Year 6	
solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate	*this will not be examined, but teaching in Year 6 is advised*
use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to three decimal places	
convert between miles and kilometres	*this will not be examined, but teaching in Year 6 is advised*
recognise that shapes with the same areas can have different perimeters and vice versa	

National Curriculum descriptors	11+ examination guidance
recognise when it is possible to use formulae for area and volume of shapes	*candidates should recognise when it is possible to use formulae to calculate the areas of right-angled triangles, including standard units of cm² and m², otherwise this will not be examined, but teaching in Year 6 is advised*
calculate the area of parallelograms and (non right-angled) triangles	only area of right-angled triangles *will* be examined, but teaching of areas generally in Year 6 is advised
calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres (cm^3) and cubic metres (m^3), and extending to other units [for example, mm^3 and km^3]	find the volume of shapes by counting cubes and know the unit cm^3 *will* be examined
Geometry – properties of shape Pupils should be taught to:	
Year 5	
identify 3-D shapes, including cubes and other cuboids, from 2-D representations	
know angles are measured in degrees; estimate and compare acute, obtuse and reflex angles	
draw given angles, and measure them in degrees (^{o})	
identify: • angles at a point and one whole turn (total 360^o) • angles at a point on a straight line and a half turn (total 180^o) • other multiples of 90^o	
use the properties of rectangles to deduce related facts and find missing lengths and angles	
distinguish between regular and irregular polygons based on reasoning about equal sides and angles	
Year 6	
draw 2-D shapes using given dimensions and angles	
recognise, describe and build simple 3-D shapes, including making nets	*this will not be examined, but teaching in Year 6 is advised*
compare and classify geometric shapes based on their properties and sizes and *find unknown angles in any triangles, quadrilaterals, and regular polygons*	*only finding angles in triangles and rectangles will be examined, but teaching in Year 6 of finding other unknown angles is advised*
illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius	*this will not be examined, but teaching in Year 6 is advised*

National Curriculum descriptors	11+ examination guidance
recognise angles where they meet at a point, are on a straight line, or are vertically opposite angles and angles in a triangle, and find missing angles	
Geometry – position and direction Pupils should be taught to:	
Year 5	
identify, describe and represent the position of a shape following a reflection or translation, using the appropriate language, and know that the shape has not changed	recognise that shapes with the same areas can have different perimeters and vice versa draw 2D shapes using given dimensions and angles
Year 6	
describe positions on the full coordinate grid (all four quadrants)	only describing and plotting positions on a 2D grid as coordinates in the first quadrant *will* be examined, but teaching in Year 6 is advised
draw and translate simple shapes on the coordinate plane, and *reflect them in the axes*	*reflecting in coordinate axes will not be examined, but teaching in Year 6 is advised*
Statistics Pupils should be taught to:	
Year 5	
solve comparison, sum and difference problems using information presented in a line graph	
complete, read and interpret information in tables, including timetables	to include information presented in bar charts, pictograms, tables, line graphs and time graphs the collection and recording of discrete data including in grouped frequency tables, to include tallying
Year 6	
interpret *(and construct)* pie charts and line graphs and use these to solve problems	candidates will be expected to interpret pie charts, including connecting their work on angles, fractions, and percentages; *the construction of pie charts will not be examined but teaching in Year 6 is advised*
calculate and interpret the mean as an average	data limited to discrete values in lists as frequency tables (but not grouped frequency tables)

13+ SYLLABUS CONTENT

In the syllabus below:

- the complete key stage 3 programme of study is shown in the left hand column
- elements of the programme *shown in italics* are expected to be taught during Year 9 and will **not** be examined at Common Entrance
- some elements of the programme not shown in italics will only be examined at Level 3 and are not expected to be taught to Level 1 and Level 2 candidates
- any distinction between what might be examined at Level 1, Level 2, Level 3 or in the Common Academic Scholarship examination, either in content or difficulty, is described in the respective columns
- preparatory schools may choose to teach additional topics, but should aim for understanding rather than quantity of material
- for more detailed information, prep schools are advised to look at the *National Curriculum in England: Mathematics Programmes of Study* document, https://www.gov.uk/government/publications/national-curriculum-in-england-mathematics-programmes-of-study published in September 2013 and updated in July 2014, including non-statutory notes and guidance

National Curriculum descriptors	Level 1	Level 2	Level 3 and CASE
Number Pupils should be taught to:			
understand and use place value for decimals, measures and integers of any size			
order positive and negative integers, decimals and fractions; use the number line as a model for ordering of the real numbers; use the symbols =, ≠, <, >, ≤, ≥			
use the concepts and vocabulary of prime numbers, factors (or divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation *and the unique factorisation property*	HCF and LCM may be tested implicitly	HCF and LCM may be tested implicitly	HCF and LCM may be tested explicitly
use the 4 operations, including formal written methods, applied to integers, decimals, proper and improper fractions, and mixed numbers, all both positive and negative	calculations with negative numbers will be restricted to integers; mixed numbers will be simple e.g. 1⅓	calculations with negative numbers will be restricted to integers; mixed numbers will be familiar e.g. 7½ or 2¾	calculations will be with the full range of numbers
use conventional notation for the priority of operations, including brackets, powers, roots and reciprocals	reciprocals will be described in the form $\frac{1}{x}$ and not x^{-1}	reciprocals will be described in the form $\frac{1}{x}$ and not x^{-1}	reciprocals will be described in the form $\frac{1}{x}$ and not x^{-1}
recognise and use relationships between operations including inverse operations	inverse operations will be numerical only	inverse operations will be numerical only	inverse operations will be numerical only

National Curriculum descriptors	Level 1	Level 2	Level 3 and CASE
use integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5 and distinguish between exact representations of roots and their decimal approximations	square roots and very simple powers e.g. 2^4 may be tested	square and cube roots and powers up to 5 may be tested	square and cube roots and powers up to 5 may be tested; candidates should understand that $\sqrt{2} \neq 1.4$
interpret and compare numbers in standard form A x 10^n $1 \leq A < 10$, where n is a positive or negative integer or 0	standard form will not be examined	standard form will not be examined	standard form may be examined
work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $\frac{7}{2}$ or 0.375 and $\frac{3}{8}$)	simple numbers only		
define percentage as 'number of parts per hundred', interpret percentages and percentage changes as a fraction or a decimal, interpret these multiplicatively, express 1 quantity as a percentage of another, compare 2 quantities using percentages, and work with percentages greater than 100%	percentages will be less than 100%	percentages greater than 100% will be simple, e.g. 125%	
interpret fractions and percentages as operators			
use standard units of mass, length, time, money and other measures, including with decimal quantities			
round numbers and measures to an appropriate degree of accuracy [for example, to a number of decimal places or significant figures]	decimal places only precise instructions will be given to candidates	precise instructions will be given to candidates	precise instructions will be given to candidates
use approximation through rounding to estimate answers *and calculate possible*			

resulting errors expressed using inequality notation $a<x\leq b$			
National Curriculum descriptors	**Level 1**	**Level 2**	**Level 3 and CASE**
use a calculator and other technologies to calculate results accurately and then interpret them appropriately	+, - , × , ÷ , 1/*x* , x^2 , x^3 and π functions are required		
appreciate the infinite nature of the sets of integers, real and rational numbers			
Algebra Pupils should be taught to:			
use and interpret algebraic notation, including:			
• *ab* in place of *a* × *b*			
• 3*y* in place of *y* + *y* + *y* and 3 × *y*			
• a^2 in place of *a* × *a*, a^3 in place of *a* × *a* × *a*; a^2b in place of *a* × *a* × *b*			
• $\frac{a}{b}$ in place of *a* ÷ *b*			
• coefficients written as fractions rather than as decimals			
• brackets			
substitute numerical values into formulae and expressions*, including scientific formulae*	no understanding of a scientific formula will be required	no understanding of a scientific formula will be required	no understanding of a scientific formula will be required
understand and use the concepts and vocabulary of expressions, equations, inequalities, terms and factors	inequalities will not be examined	inequalities will not be examined	inequalities may be examined

National Curriculum descriptors	Level 1	Level 2	Level 3 and CASE
simplify and manipulate algebraic expressions to maintain equivalence by:			
• collecting like terms			
• multiplying a single term over a bracket	the term outside the bracket will be restricted to a single integer such as 3(*x*-1)	the term outside the bracket will be restricted to a single integer such as 3(2*x*-1)	the term outside the bracket may include an algebraic term and may include expansions of the form 3*x*(2*x* -1)
• taking out common factors	the factor will be restricted to an integer	the factor will be restricted to an integer	the factor may include an algebraic term
• dividing an expression by an integer or by another expression	the divisor will be restricted to an integer, such as $\frac{6x}{2}$	the divisor will be restricted to an integer, such as $\frac{6(x+3)}{2}$	the divisor may be algebraic, such as $\frac{6x(3x-2)}{2x}$
• *expanding products of 2 or more binomials*			
understand and use standard mathematical formulae*; rearrange formulae to change the subject*	simple formula and simple substitution to find the subject of the formula	substitution to find the subject of the formula	substitution to find an element of the formula; re-arrangement of the numbers may be required
model situations or procedures by translating them into algebraic expressions or formulae and by using graphs	a simple linear equation may be formed and solved	a linear equation may be formed and solved	simultaneous linear equations may be formed and solved

National Curriculum descriptors	Level 1	Level 2	Level 3 and CASE
use algebraic methods to solve linear equations in 1 variable (including all forms which require rearrangement)	candidates may be asked to solve equations such as: $x + 5 = 12$ $3x = 12$ $3x - 4 = 8$ $3(x - 2) = 12$ $\frac{x}{4} = 3$	candidates may be asked to solve equations such as: $7 = 3x + 5$ $3x - 4 = 8 - x$ $3(5 + 2x) = 21$ $\frac{1}{3}(x - 4) = 2$	candidates may be asked to solve equations such as: $4 - 3x = 9$ $\frac{4}{3}x - 2 = 10$ $\frac{2}{3}(2x - 1) = 6$
use algebraic methods to solve linear equations in 2 variables *(including all forms which require rearrangement)*	this will not be examined	this will not be examined	simultaneous equations may be examined pairs of simultaneous equations of the form $y = 2x + 3$ and $y = 5x - 9$ or $2x + 3y = 5$ and $4x - 2y = 18$ but substitution of one equation into the other only in very simple cases such as $y = 2x$ and $3x + 2y = 21$
work with coordinates in all 4 quadrants	coordinates will have integer values only	coordinates will have integer values only, but decimal answers may be read from graphs	

National Curriculum descriptors	Level 1	Level 2	Level 3 and CASE
recognise, sketch and produce graphs of linear and quadratic functions of 1 variable with appropriate scaling, using equations in *x* and *y* and the Cartesian plane	$x = \pm c$, $y = \pm c$, $y = \pm x$ only	graphs of linear functions only	graphs of linear and quadratic functions
interpret mathematical relationships both algebraically and graphically			
reduce a given linear equation in two variables to the standard form y = mx + c; calculate and interpret gradients and intercepts of graphs of such linear equations numerically, graphically and algebraically			

National Curriculum descriptors	Level 1	Level 2	Level 3 and CASE
use linear and quadratic graphs to estimate values of *y* for given values of *x* and vice versa and to find approximate solutions of simultaneous linear equations	work only on linear graphs will be examined, e.g. a conversion graph; finding solutions graphically to simultaneous linear equations will not be examined	work only on linear graphs will be examined; the use of intersecting graphs to find solutions to simultaneous linear equations will only be implicit	work on two linear graphs or one linear and one quadratic graph may be examined; finding solutions graphically to simultaneous equations may be tested at Level 3
find approximate solutions to contextual problems from given graphs of a variety of functions, including piece-wise linear, *exponential and reciprocal* graphs	conversion graphs or very simple distance-time graphs may be examined	conversion graphs or distance-time graphs may be examined	conversion graphs or distance-time graphs may be examined
generate terms of a sequence either from a term-to-term or a position-to-term rule	these will be straightforward especially if they obey a position-to-term rule		
recognise arithmetic sequences and find the *n*th term	recognition of simple patterns only	recognition of simple patterns only but expression for *n*th term useful	*n*th term required
recognise geometric sequences and appreciate other sequences which arise	recognition of how simple numerical patterns develop	recognition of how simple numerical patterns develop	recognition of numerical patterns; any algebraic work on sequences is likely to be restricted to the Scholarship

Additional Algebra topics included at Level 3 only			
National Curriculum descriptors	**Level 1**	**Level 2**	**Level 3 and CASE**
solving simple linear inequalities	this will not be examined	this will not be examined	inequalities of the form $2(x - 3) < 9$ or $5 > 3 - 2x$
solving simple quadratic equations	this will not be examined	this will not be examined	equations such as $x^2 = 9$ or $x(x + 2) = 2x + 16$
Ratio, proportion and rates of change Pupils should be taught to:			
change freely between related standard units [for example time, length, area, volume/capacity, mass]			
use scale factors, scale diagrams and maps	scale factors may be either in the form '1:50000' or '1 cm represents 500m'.	scale factors may be either in the form '1:50000' or '1 cm represents 500m'	scale factors may be either in the form '1:50000' or '1 cm represents 500m'
express 1 quantity as a fraction of another, where the fraction is less than 1 and greater than 1	fractions will be straightforward especially if they are greater than 1		
use ratio notation, including reduction to simplest form			
divide a given quantity into 2 parts in a given part:part or part:whole ratio; express the division of a quantity into 2 parts as a ratio		division into 3 parts included	division into 3 parts included

understand that a multiplicative relationship between 2 quantities can be expressed as a ratio or a fraction		division into 3 parts included	division into 3 parts included
relate the language of ratios and the associated calculations to the arithmetic of fractions and to linear functions			

National Curriculum descriptors	Level 1	Level 2	Level 3 and CASE
solve problems involving percentage change, including: percentage increase, decrease and original value problems and simple interest in financial mathematics	original value problems will not be examined simple interest will be implied	original value problems will not be examined simple interest will be implied	original value problems may be examined simple interest will be implied
solve problems involving direct *and inverse* proportion, including graphical representations			
use compound units such as speed, unit pricing and density to solve problems	speed may be examined in simple terms; the relationship between distance, time and speed will be useful but not essential density and unit pricing will be implicit	speed may be examined density and unit pricing will be implicit	the average speed for multi-stage journeys may be examined density and unit pricing will be implicit
Geometry and measures Pupils should be taught to:			
derive and apply formulae to calculate and solve problems involving: perimeter and area of triangles, parallelograms, *trapezia*, volume of cuboids (including cubes) and other prisms (including cylinders)	the volume of prisms will not be examined	the volume of prisms will not be examined	the volume of prisms may be examined
calculate and solve problems involving: perimeters of 2-D shapes (including circles), areas of circles and composite shapes	the radius or diameter will be given; the shapes will be simple	the radius or diameter will be given	candidates may be expected to find the radius or diameter of a circle with a given circumference or area

National Curriculum descriptors	Level 1	Level 2	Level 3 and CASE
draw and measure line segments and angles in geometric figures, including interpreting scale drawings	construction of simple plane figures; simple three-figure bearings and points of the compass	construction of plane figures; three-figure bearings and points of the compass	construction of plane figures; three-figure bearings and points of the compass
derive and use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle); recognise and use the perpendicular distance from a point to a line as the shortest distance to the line	simple constructions only with no derivation required	simple constructions which may be used to help draw or find the area of a plain figure; an explanation may be asked for	simple constructions which may be used to help draw or find the area of a plain figure; an explanation may be asked for
describe, sketch and draw, using conventional terms and notations: points, lines, parallel lines, perpendicular lines, right angles, regular polygons, and other polygons which are reflectively and rotationally symmetric	regular polygons to be limited to equilateral triangle, square, hexagon and octagon	regular polygons will be limited to those whose exterior angle is an integral number of degrees the names of polygons up to decagons will be expected	
use the standard conventions for labelling the sides and angles of triangle ABC*, and know and use the criteria for congruence of triangles*			
derive and illustrate properties of triangles, quadrilaterals, circles, and other plane figures [for example, equal lengths and angles] using appropriate language *and technologies*	simple figures only radius/diameter relationship and simple fractions of circles only	arcs and sectors of circles may be examined	arcs and sectors of circles may be examined as well as prisms derived from them
identify properties of, and describe the results of, translations, rotations and reflections applied to given figures	vector terminology will not be examined; translations will be described in terms of units moved left/right and up/down	vector terminology will not be examined; translations will be described in terms of units moved left/right and up/down	vector terminology will not be examined; translations will be described in terms of units moved left/right and up/down

National Curriculum descriptors	Level 1	Level 2	Level 3 and CASE
identify and construct congruent triangles, and construct similar shapes by enlargement, with and without coordinate grids	candidates may be asked to construct a triangle, but will not be examined on the congruency of triangles enlargements will be by a positive, integral scale factor	candidates may be asked to construct a triangle, but will not be examined on the congruency of triangles enlargements will be by a positive, integral scale factor candidates should know the relationship between scale factor and area factor	candidates may be asked to construct a triangle, but will not be examined on the congruency of triangles enlargements may be by a positive fractional scale factor, and candidates may be required to deduce the scale factor and centre of enlargement
apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles			
understand and use the relationship between parallel lines and alternate and corresponding angles	the language used will be straightforward		
derive and use the sum of angles in a triangle and use it to deduce the angle sum in any polygon, and to derive properties of regular polygons	the angle sum of polygons with up to 4 sides may be examined candidates may be examined at all levels on the symmetry properties of regular polygons	the angle sum of polygons with up to 5 sides may be examined candidates may be examined at all levels on the symmetry properties of regular polygons	the angle sum of polygons with more than 5 sides may be examined candidates may be examined at all levels on the symmetry properties of regular polygons
apply angle facts, *triangle congruence, similarity* and properties of quadrilaterals to derive results about angles and sides, including Pythagoras' Theorem, and use known results to obtain simple proofs	Pythagoras' Theorem will not be examined	Pythagoras' Theorem will not be examined	Pythagoras' Theorem may be examined but not derived

National Curriculum descriptors	Level 1	Level 2	Level 3 and CASE
use Pythagoras' Theorem *and trigonometric ratios in similar triangles* to solve problems involving right-angled triangles	Pythagoras' Theorem will not be examined	Pythagoras' Theorem will not be examined	Pythagoras' Theorem may be examined but not derived
use the properties of faces, surfaces, edges and vertices of cubes, cuboids, prisms, cylinders, pyramids, *cones and spheres* to solve problems in 3-D	prisms, cylinders and pyramids will not be examined	candidates may be asked to draw the net of a prism (but not a cylinder) or pyramid and use it to calculate the surface area	candidates may be asked to calculate the surface area or volume of a prism and the surface area of a pyramid from its net
interpret mathematical relationships both algebraically and geometrically			
Probability Pupils should be taught to:			
record, describe and analyse the frequency of outcomes of simple probability experiments involving randomness, fairness, equally and unequally likely outcomes, using appropriate language and the 0-1 probability scale			
understand that the probabilities of all possible outcomes sum to 1			
enumerate sets and unions/intersections of sets systematically, using tables, grids and Venn diagrams	tables and grids may be examined; sets and unions/intersections will be implicit; Venn diagrams will not be mentioned	tables and grids may be examined; sets and unions/intersections will be implicit; Venn diagrams will not be mentioned	tables and grids may be examined; sets and unions/intersections will be implicit; Venn diagrams will be mentioned only in the Scholarship paper
generate theoretical sample spaces for single and combined events with equally likely, mutually exclusive outcomes and use these to calculate theoretical probabilities	combined events will be limited to two events, the outcomes of which will be listed in a table	combined events will be limited to three events, the outcomes of which will be listed in a table	combined events will be limited to three events, the outcomes of which will be listed in a table

National Curriculum descriptors	Level 1	Level 2	Level 3 and CASE
Statistics Pupils should be taught to:			
describe, interpret and compare observed distributions of a single variable through: appropriate graphical representation involving discrete, *continuous and grouped* data; appropriate measures of central tendency (mean, mode, median) and spread (range, consideration of outliers)	all data will be discrete consideration of outliers will be limited to understanding that a measurement is unusual	all data will be discrete consideration of outliers will be limited to understanding that a measurement is unusual	all data will be discrete consideration of outliers will be limited to understanding that a measurement is unusual
construct and interpret appropriate tables, charts, and diagrams, including frequency tables, bar charts, pie charts, and pictograms for categorical data, and vertical line (or bar) charts for ungrouped *and grouped* numerical data			
describe simple mathematical relationships between 2 variables (bivariate data) in observational and experimental contexts and illustrate using scatter graphs	candidates may be asked to describe the relationship between two variables on a scatter graph, and draw by eye a straight 'line of best fit' and interpret that line	candidates may be asked to describe the relationship between two variables on a scatter graph, and draw by eye a straight 'line of best fit' and interpret that line	candidates may be asked to describe the relationship between two variables on a scatter graph, and draw by eye a straight 'line of best fit' and interpret that line

APPENDIX 1

NATIONAL CURRICULUM KEY STAGE 3 TOPICS NOT EXAMINED AT 13+
(RECOMMENDED FOR TEACHING IN YEAR 9)

Number

- unique factorisation properties
- HCF and LCM (except for Level 3 and Common Academic Scholarship)
- calculation of possible errors as a result of rounding, expressed using inequality notation $a < x \leq b$
- standard form (except for Level 3 and Common Academic Scholarship)
- appreciation of the infinite nature of the sets of integers, real and rational numbers

Algebra

- understanding of scientific formulae
- understanding and use of inequalities (except for Level 3 and Common Academic Scholarship)
- expansion of products of 2 or more binomials
- rearrangement of formulae to change the subject
- use of algebraic methods to solve linear equations in two variables (except for Level 3 and Common Academic Scholarship)
- reduction of a given linear equation in two variables to the standard form $y = mx + c$; calculation and interpretation of gradients and intercepts of graphs of linear equations numerically, graphically and algebraically
- quadratic graphs (except for Level 3 and Common Academic Scholarship)
- exponential and reciprocal graphs
- formulae for quadratic sequences
- solving of simple quadratic equations (except for Level 3 and Common Academic Scholarship)

Ratio, proportion and rates of change

- solving of problems involving inverse proportion and algebraic representation of direct and inverse proportion

Geometry and measures

- derivation of formulae for the perimeter and area of triangles, parallelograms, trapezia, volume of cuboids and other prisms; use of the formula for the area of a trapezium; volume of prisms (except for Level 3 and Common Academic Scholarship)
- radius or diameter of a circle given the circumference or area (except for Level 3 and Common Academic Scholarship)

- knowledge and use of the criteria for congruence of triangles
- derivation of properties of triangles, quadrilaterals, circles and other plane figures, using appropriate technologies
- application of triangle congruence and similarity of quadrilaterals to derive results about angles and sides
- use of trigonometric ratios in similar triangles to solve problems involving right-angled triangles
- use of the properties of cones and spheres to solve problems in 3-D

Probability

- Venn diagrams (except for Common Academic Scholarship)

Statistics

- graphical representation and measures of central tendency and spread involving continuous and grouped data
- calculation of outliers
- construction and interpretation of tables, charts and diagrams for grouped numerical data